A Novel Chaotic System

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I present a magnetic device that generates a chaotic signal. A sphere with many embedded magnets is levitated on an air bearing, and its motion is excited by randomly pulsed surrounding electromagnets. A magnetic sensor in proximity to the surface of the sphere generates a chaotic voltage signal produced by the motion of the sphere.

Introduction

While science has discovered a plethora of chaotic systems in nature, there are just a few simple mechanical devices that produce chaotic motion. Examples of these are the double pendulum, the swinging Atwood's machine, and the elastic pendulum. Doubochinski and colleagues demonstrated the unusual, but not chaotic, motion of a pendulum with an affixed magnet when it is subjected to the force of a stationary electromagnet excited by an oscillating electric current.[1-2]

In this study, I go beyond this magnetic pendulum system having motion constrained to a two-dimensional surface, to the three-dimensional motion of magnets in a floating sphere. The motion of the sphere is influenced by a trio of surrounding electromagnets randomly pulsed. The motion of the sphere was analyzed, and the voltage signal from a magnetic sensor in proximity to the surface of the sphere was found to be chaotic.

Sphere

A 2-3/4 inch diameter dense Styrofoam sphere was selected as a host for the embedded magnets. An air bearing was created by casting a polyurethane molding material inside a section of four inch diameter PVC pipe to conform to the shape of the lower portion of the sphere, which was coated with thin layer of petroleum jelly mold-release agent. The less dense Styrofoam will float on the surface of the polyurethane and will not immerse to the required depth for casting without proper fixture.

After molding, the polyurethane basin was perforated with a regular array of twelve holes drilled with a #60 (0.040" diameter) drill to allow air flow from the PVC plenum to levitate the sphere.

To facilitate the random placement of magnets in the sphere, a computer program was written to create a gore (fig. 1) that was attached to the surface of the sphere with a removable adhesive. The random pattern was created to have each magnet paired with another on the opposite side of the sphere. This keeps the center of gravity at the center of the sphere. A minimum spacing between magnets was required.

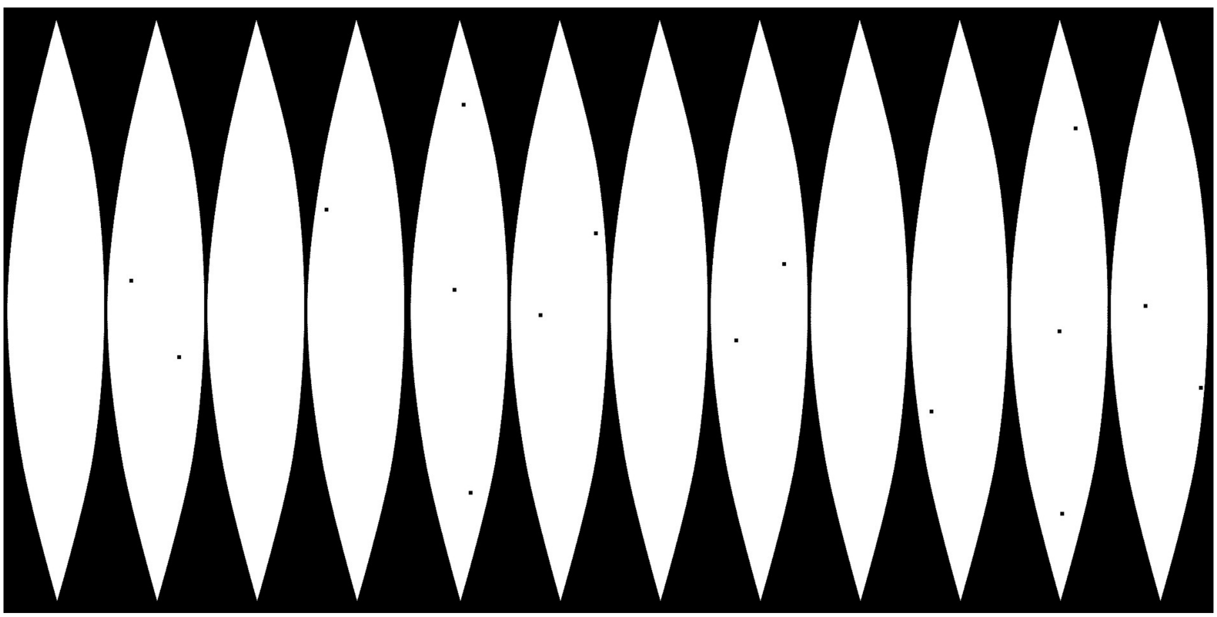

Fig. 1 . Gore pattern used to facilitate drilling of holes for magnet placement in the sphere. Each hole is paired with another on the opposite side of the sphere.

The magnets were nickel-plated neodymium-iron-boride, 0.187 inch diameter by 0.25 inch length (Part No. N42P187250, Bunting, 1150 Howard Street, Elk Grove Village, IL 60007). Holes of 0.187 inch diameter were drilled to a depth of about 0.3 inch at the specified points, the gore was removed, and the magnets were placed in the holes with an adhesive agent. Care was taken to ensure that the magnets were placed with the same polarity directed outwards from the sphere.

The remaining space of the holes was filled with a spackle material that was also used to smooth the surface of the sphere. The surface was polished with fine sandpaper, and the sphere was floated in water to examine any deviation of the center of gravity from the center of the sphere. The center of gravity was adjusted by insertion of small lengths of AWG 12 copper wire into the sphere at the proper locations. The small holes created by such insertion were filled with spackle. A random pattern was applied to the surface of the sphere with an inked stamp to aid detection of sphere motion. The sphere is shown in fig. 2.

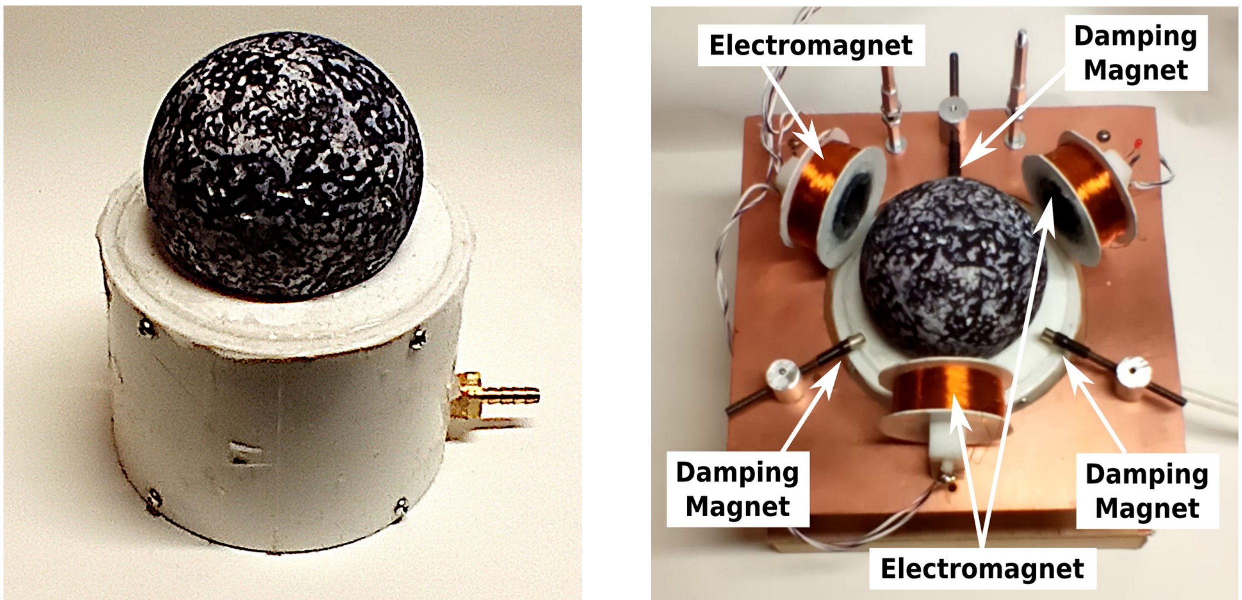

Fig. 2. Left, the finished sphere seated in its air bearing. Right, placement of the electromagnets. Also shown are the three damping magnets described in the text. Air was supplied by a Porter Cable Model C2002 Portable Electric Pancake Air Compressor with an airflow of about 3 cubic feet per minute.

<u>Electromagnetic Excitation</u>

The floating sphere was surrounded by three electromagnets, as shown in fig. 2. The magnets were wound from 500 turns of AWG 30 enameled copper wire on a one inch O.D. PVC coil form one inch in length. The coils had a resistance of slightly more than 25-ohms, which gave a current of about 0.5 ampere at their 12 volt excitation. The interior of each coil form was filled with a stack of two ferrite toroids 1-1/8 inch in diameter, 3/16 inch thick, with a central 3/4 inch hole.

The electromagnets were excited by random pulses produced by a software simulation of a 16 bit maximal length linear feedback shift register, as shown in fig. 3. The bit taps at (A,B,C) produce a number from 0-7, and this number was used to excite the electromagnets in a particular manner. States 0 (no electromagnets energized) and state 7 (all electromagnets energized) were disallowed states, and the LFSR was cycled to obtain a new value. The other states energized one or two electromagnets at a time using the following logic table.

Table I. Logic table for electromagnet excitation.

LFSR Output	Electromagnet 1	Electromagnet 2	Electromagnet 3
1	Energized	Off	Off
2	Off	Energized	Off
3	Energized	Energized	Off
4	Off	Off	Energized
5	Energized	Off	Energized
6	Off	Energized	Energized

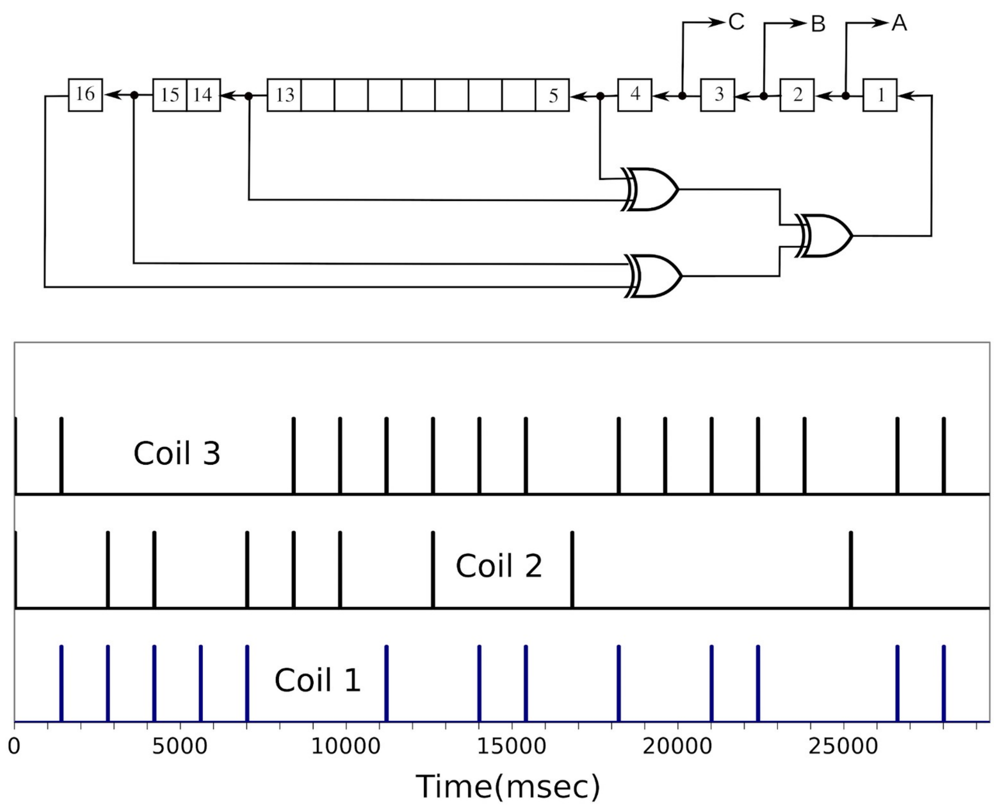

Fig. 3. Top, the linear feedback shift register used for generation of pseudorandom pulse excitation of the electromagnets. Bottom, a portion of the excitation sequence.

The electromagnet random pulse excitation was accomplished by the circuitry in fig. 4. The pulse width and pulsing period are controlled by two potentiometers. The range of settings are 10-110 msec for pulse width and 200-2200 msec for pulse period. In the examples in this paper, the pulse width was 30 msec, and the pulse period was 900 msec. The polarity of the electromagnetic field was selected to repulse the embedded magnets of the sphere.

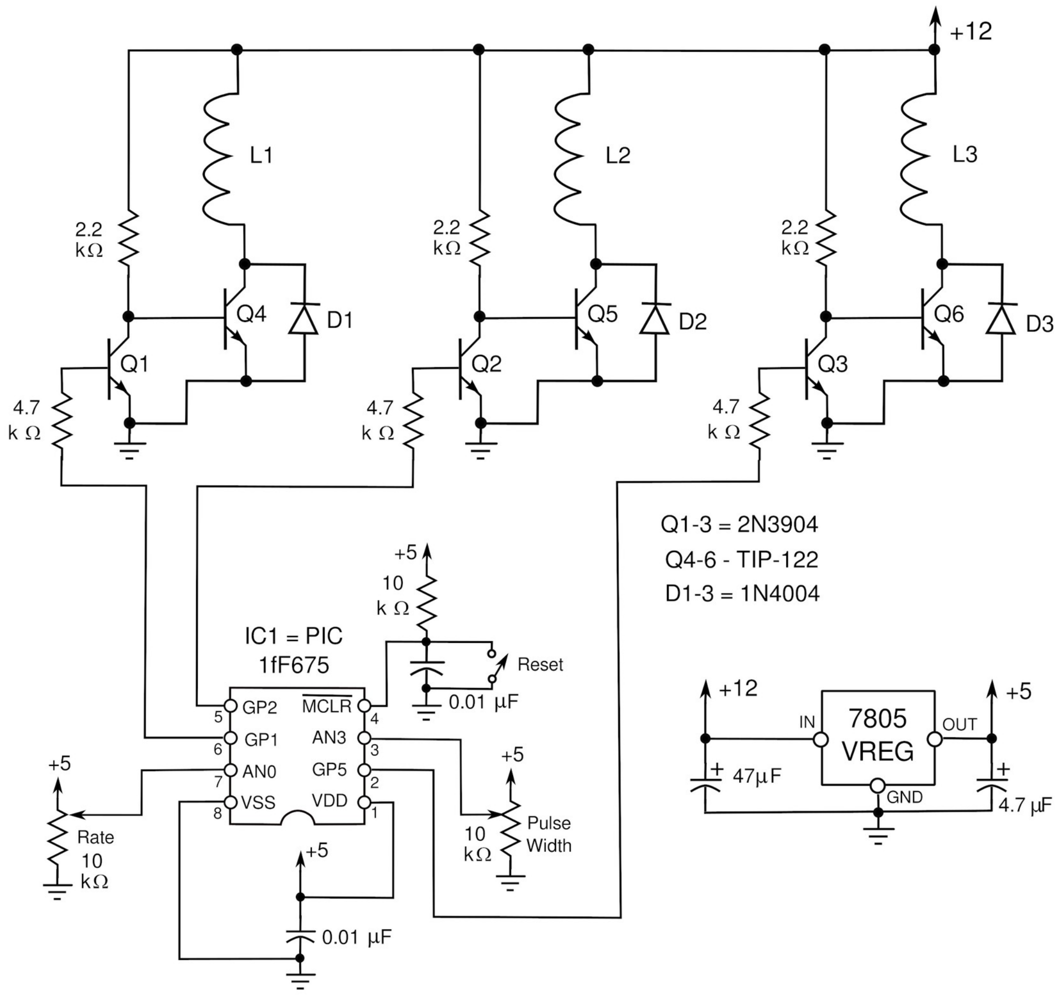

Fig. 4. Electromagnet driver circuitry. The linear feedback shift register and control logic are implemented in the PIC 16F675. The electromagnets are shown as L1-L3. Source and object code for the microcontroller are available by request to the author.

Analysis of Sphere Motion

It was found that the sphere tended to lock into an orbit with a periodic precession without damping, possibly because of an imbalance in the air flow of the air bearing. Damping was accomplished by placement of three repulsing permanent magnets, as shown in fig. 2, with the damping adjusted by distance. The magnets used were the same type as those embedded in the sphere.

Sensing of the movement of the sphere was attempted using machine vision techniques, but it was found that the 30 fps frame rate of available cameras was too slow to capture the motion at its most rapid points. It's estimated that a frame rate an order of magnitude larger would be required. Instead, the optical flow sensor from an optical mouse, placed at a 2 mm offset from the surface of the sphere, was used to extract the incremental movement of the sphere in "x" and "y" directions that roughly correspond to the *theta* and *phi* angular movement of the sphere. Some representative portions of this motion are shown in fig. 5.

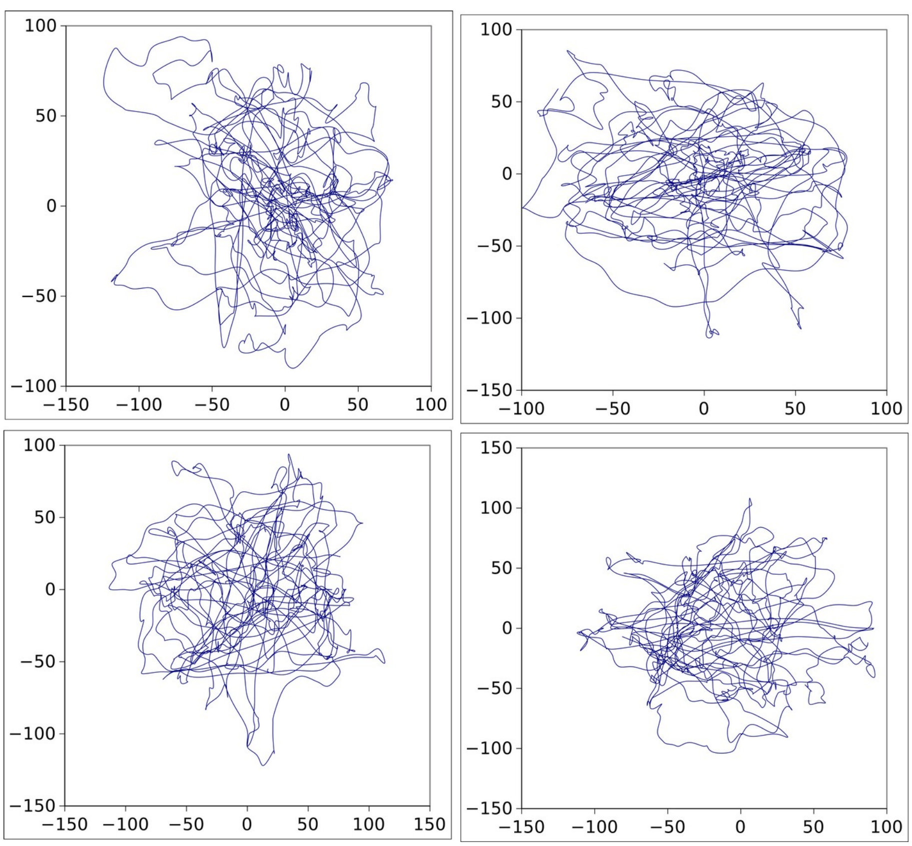

Fig. 5. Sphere motion as detected by the optical sensor of an optical mouse. The sensor gives incremental motion in "x" and "y" directions in a range of ±127.

Magnetic Field Sensing

A GMR bridge sensor (Part no. AA007-00E from NVE Corporation, Eden Prairie, MN) was place at a 3 mm distance from the surface of the sphere. This sensor is omnipolar, so it responds positively to both magnetic polarities with a field sensing direction parallel to the face of the sensor package. Under five volt excitation, it has a sensitivity of 0.5 mV/Oe. The amplified voltage corresponding to the magnetic field was logged at half-second intervals. In this manner, a time series of 7,000 samples of the sensed magnetic field was obtained in somewhat less than an hour. A small portion of this time series is shown in fig. 6.

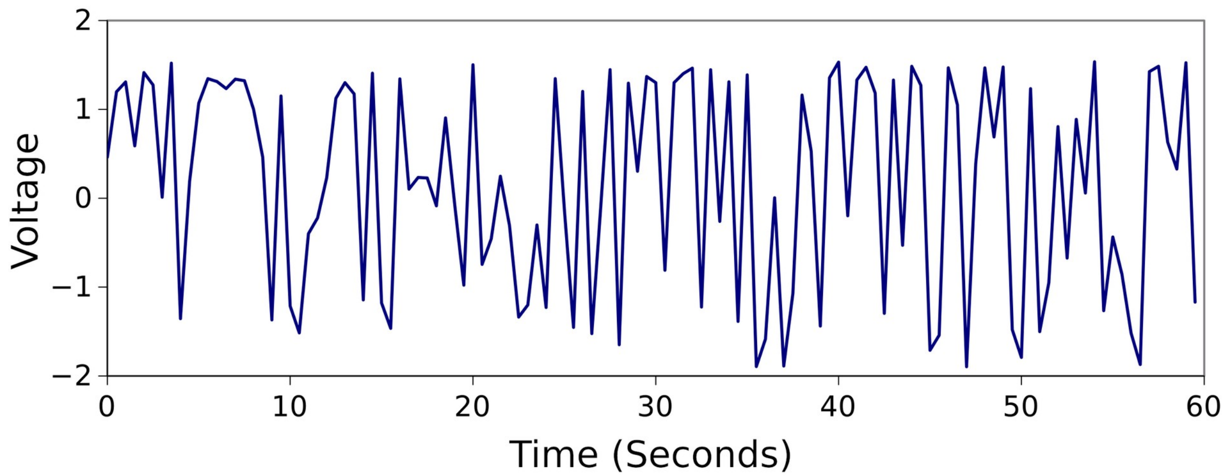

Fig. 6. Voltage corresponding to the sensed magnetic field. The actual voltage signal is unipolar. In this representation, the voltage is subtracted from the mean value of the entire time series to show a bipolar response.

The chaotic nature of this time series was analyzed using the DChaos package of the R statistical analysis program. The 7,000 sample series gave a maximum Lyapunov exponent of -0.7138 with a standard error of 0.0202, a result that indicates a chaotic system. The R analysis used the norma-2 by bootstrap blocking method with an embedding dimension of 3, a time-delay of 1, 2 hidden units, and a block length of 110 for 100 blocks.

Discussion

This device can be miniaturized, and an array of many spheres can be levitated on air bearing cavities on a single air plenum. The spheres can be contained between upper and lower air bearings to allow movement without dislodge of the spheres. The chaotic signals of such an array can possibly be combined to create high quality random numbers for cryptography.

References

1. Ya.B. Duboshinski, L.A. Vainshtein, Excitation of low-frequency oscillations by a high-frequency force, Zh. Tech. Fiz. 48, 1321 (1978) [Sov. Phys.-Tech. Phys. 23, 745 (1979)].
2. D.B. Doubochinski, Ya.B. Duboshinski, A.S. Magarchak, V.Chabanski, Discrete modes of a system subject to an inhomogeneous high-frequency force, Zh. Tech. Fiz. 49, 1160 (1979) [Sov. Phys.-Tech. Phys. 24, 642 (1979)].